Bergenfield High School

Bergenfield, New Jersey

Mathematics Department

Summer Course Work

in preparation for

Algebra II

Completion of this summer work

is required on the first day of the

2023 - 2024 school year.

Student Name:_____

June 2023 ,

Dear Parents and Guardians:

We are excited to present summer activities that math teachers of Bergenfield High School have created. Enclosed are math activities designed to help your son or daughter practice the skills which they have already learned and are critical to success in this course. As you may be aware, studies have shown that students who do not practice or review during the summer month the material they have already mastered lose some of that mastery. Unfortunately, this then requires the next teacher to spend valuable teaching time reviewing. While certainly not the final answer, this packet of activities is designed to help your son or daughter retain his or her math skills and knowledge.

Like you, we want your child to enjoy a wonderful summer. We urge you to encourage your child tot take this task seriously and complete is successfully. Together we can make a difference in your child's future. Now is the time to build on the foundation to help your child succeed on future standardized exams such as the NJSLA, and even more importantly, the SAT.

These activities will reinforce skills that were taught in previous courses. This assignment is voluntary and will not be graded but is recommended to ensure success in this course. Calculuators are NOT to be used to complete this project except where noted. Please read all directions carefully.

We wish you a wonderful and safe summer.

Sincerely,

Jim Fasano   Steven Neff
Principal   Director of Mathematics

Bergenfield High School
Mathematics Department

Summer Workbook
Algebra II
Topics

1.   Algebraic Expressions

2.   Linear Equations (solving, ordered pairs, graphing)

3.   Solving Systems (graphing, substitution, elimination)

4.   Roots and Simplifying Radicals

5.   Laws of Exponents

6.   Multiplying Polynomials

7.   Solving Quadratics (factoring – GCF, a = 1, a >1; Quadratic Formula)

8. Graphing Quadratic Equations (Standard Form)

All pages MUST show the work in order for the work to be accepted. If more paper is needed, the work may go on the back of each page or neatly on a separate page.

Completion of this booklet is required by the first day of the school year.

***If you do not remember something, look it up. Use resources such as   Khan academy, google, youtube, etc.

Algebraic Expressions

PEMDAS -the order in which you evaluate expressions
P – Parenthesis
E – Exponents
M –Multiplication   (from left
D – Division   to right)
A – Addition   (from left
S — Subtraction   to right)

Evaluate each expression if $a = 2, b = -3, c = -1, and\ d = 4$

1. $2a + c$

2. $\frac{bd}{2c}$

3. $\frac{2d-a}{b}$

4. $\frac{3b}{5a+c}$

Evaluate each expression if $x = 2, y = -3, and\ z = 1$

5. $24 + |x - 4|$

6. $13 + |8 + y|$

7. $|5 - z| + 11$

8. $|2y - 15| + 7$

Linear Equations

<u>Example:</u> Solve the equation.

$$\frac{2}{3}n + 1 = 11$$

$$\frac{2}{3}n + 1 = 11$$

$$\underline{\phantom{\frac{2}{3}n} -1 \quad -1}$$

Subtract 1 from each side.

$$(3)\frac{2}{3}n = 10(3)$$

Multiply each side by 3.

$$\frac{2n}{2} = \frac{30}{2}$$

Divide each side by 2.

Solve each equation.

1. $r + 11 = 3$

2. $\frac{8}{5}a = -6$

3. $c - 14 = -11$

4. $b + 2 = -5$

5. $5t = 30$

6. $\frac{m}{10} + 15 = 21$

7. $9n + 4 = 5n + 18$

8. $-2y + 17 = -13$

9. $-2(n + 7) = 15$

Ordered Pairs

Important Notes:

- Points in the coordinate plane are known as **ordered pairs.**
- Ordered pairs are written in the form (x, y).
- The x-axis and y-axis divide the coordinate plane into four quadrants.
- The point of intersection of the axes is the **origin.**
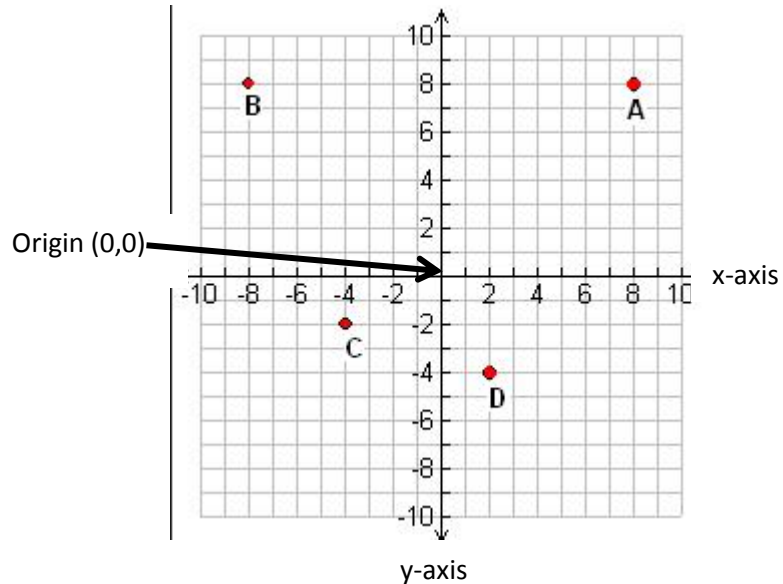- The origin is located at (0,0)

Example:

A (8, 8) – Quadrant I

B (-8, 8) – Quadrant II

C (-4, -2) – Quadrant III

D (2, -4) – Quadrant IV

Origin (0,0)

x-axis

y-axis

Write the ordered pair for each point shown in the coordinate plane. Name the quadrant that the point is located in.

1. A = (,   )

Quadrant:_____

4. B = (,   )

Quadrant: _____

2. C = (,   )

Quadrant:_____

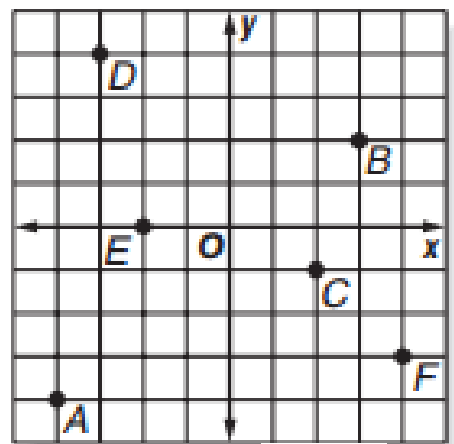5. D = (,   )
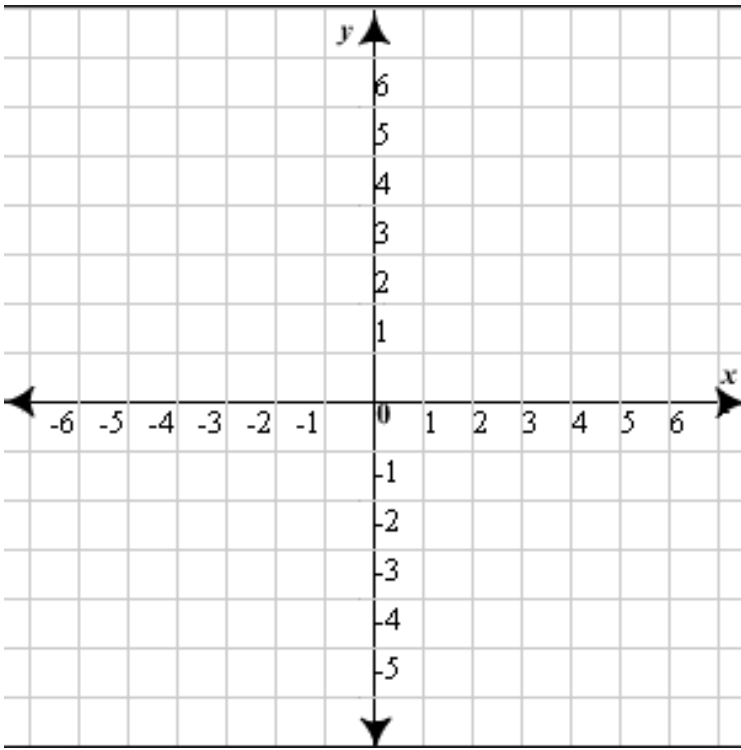
Quadrant: _____

3. E = (,   )

Quadrant:_____

6. F = (,   )

Quadrant: _____

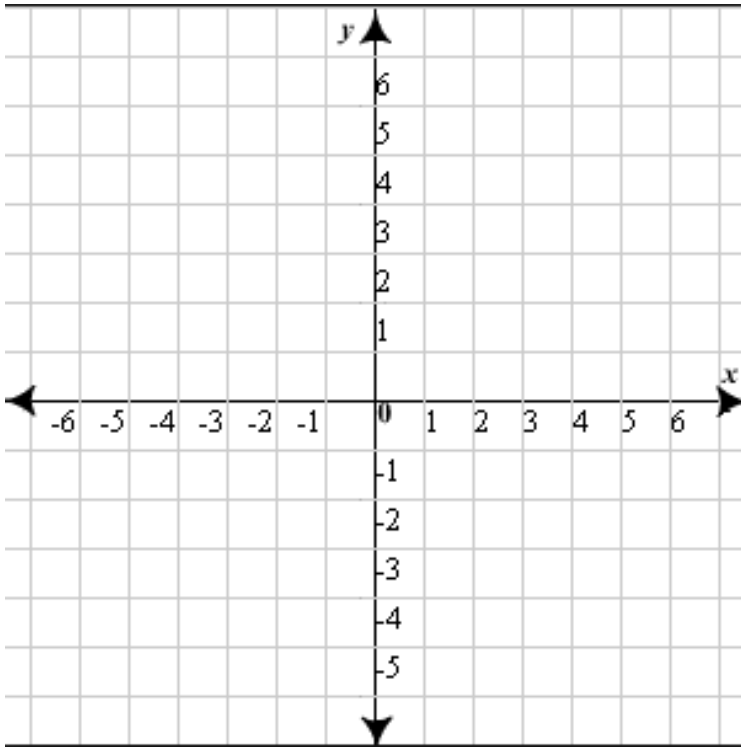Make a table of values and graph 5 points that satisfy each equation.

7. $y = 2x$

X	Y

8. $y = 4 - x$

X	Y

Systems of Linear Equations

Solving Systems by Graphing

Solution (2,3)

Example: Solve the system of equations by graphing.

$y = 2x - 1$
$x + y = 5$

Step 1: Graph each line. The second equation here needs to be changed to $y = mx + b$.

$x + y = 5$
$-x - x$
$y = -x + 5$
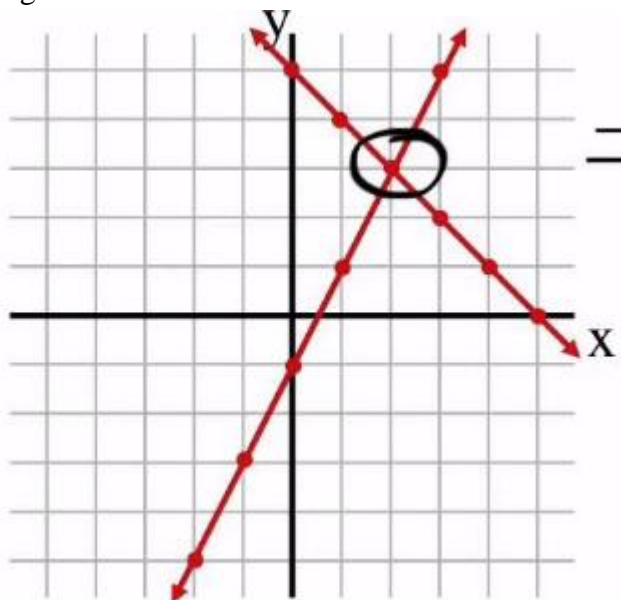
Step 2: Find the point of intersection.
If lines overlap – infinitely many solutions
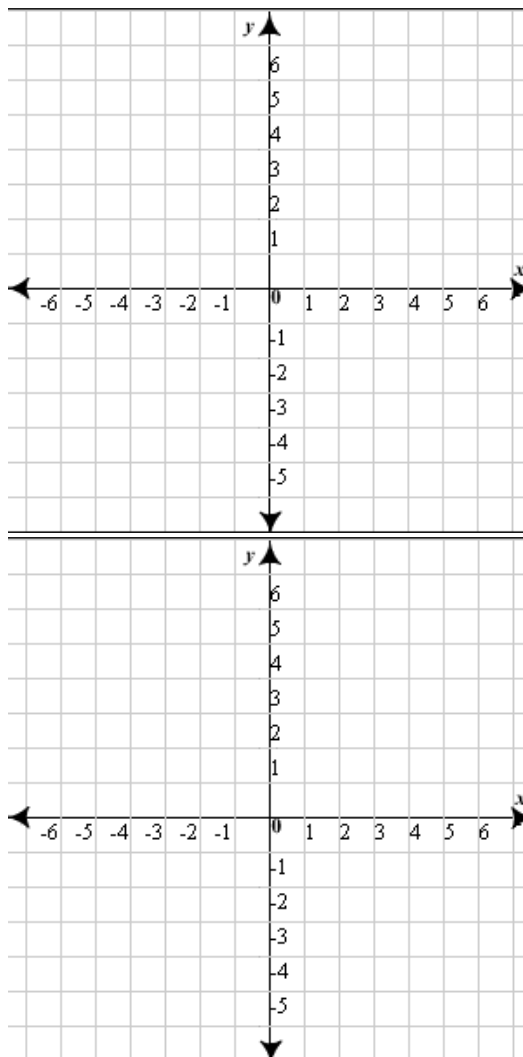If lines are parallel – no solution

Solve by graphing.

1. $y = -x + 2$
    $y = -\dfrac{1}{2}x + 1$

2. $y - 2x = 1$
    $2y - 4x = 1$

Solving Systems by Substitution

Example: Solve the system of equations by substitution.
$$y - 3x = -3$$
$$-2x - 4y = 26$$

Step 1: Solve for a variable for either equation. (It is ideal to pick the variable with a coefficient of 1)

$$y - 3x = -3$$
$$\underline{+3x \quad\quad +3x}$$
$$y = 3x - 3$$

Step 2: Plug the expression 3x − 3 in for y of the OTHER equation.
–2x – 4y = 26
–2x – 4(3x – 3) = 26

Step 3: Solve for x.
–2x – 4(3x – 3) = 26
–2x – 12x + 12 = 26
–14x + 12 = 26
–14x = 14
$x = -1$

Step 4: Plug in x for either equation to solve for y.
y = 3x – 3
y = 3(–1) – 3
y = –6

Final Solution: (– 1, – 6)

Solve by substitution.

3. $-5x + 3y = 12$
   $x + 2y = 8$

4. $x - 4y = 22$
   $2x + 5y = -21$

Solving Systems by Elimination

Example: Solve the system of equations by elimination.
$4x - 3y = 25$
$-3x + 8y = 10$

Step 1: Decide which variable you want to eliminate and find the LCM for the two coefficients for that variable.

Eliminate x → 4 and -3 have an LCM of 12

Step 2: Multiply each equation by the number that will make the x-terms have a coefficient of 12. One must be positive and the other must be negative.

$3(4x - 3y = 25)$ → $12x - 9y = 75$
$4(-3x + 8y = 10)$ $-12x + 32y = 40$

Step 3: Add the columns of like terms.
$12x - 9y = 75$
$-12x + 32y = 40$
$$23y = 115$$
$$y = 5$$

Step 4: Plug in y for either equation to solve for x.
$4x - 3y = 25$
$4x - 3(5) = 25$
$x = 10$

Final Solution: (10, 5)

Solve by Elimination.

3. $-3x + y = 7$
   $3x + 2y = 2$

4. $-4x + 5y = -11$
   $2x + 3y = 11$

Square Roots and Simplifying Radicals

- **Product Property** for two numbers a, $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

- **Quotient Property** for any numbers a and b, where a, $b \geq 0$, $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Ex. 1 Simplify $\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Ex. 3 Simplify

Ex. 2 Simplify $\sqrt{20x^3 y^5 z^6} = \sqrt{4 \cdot 5x^3 y^5 z^6} = \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6} =$

$$2\sqrt{5} \cdot x\sqrt{x} \cdot y^2\sqrt{y} \cdot z^3 = 2xy^2\sqrt{5xy}$$

Ex. 3 Simplify $\sqrt{\dfrac{25}{16}} = \dfrac{\sqrt{25}}{\sqrt{16}} = \dfrac{5}{4}$

Ex. 4 Simplify $\dfrac{2}{3-\sqrt{3}} = \dfrac{2}{3-\sqrt{3}} \cdot \dfrac{3+\sqrt{3}}{3+\sqrt{3}} = \dfrac{6+2\sqrt{3}}{9-\left(\sqrt{3}\right)^2} = \dfrac{6+2\sqrt{3}}{6} = \dfrac{3+\sqrt{3}}{3}$

Simplify the following radicals. *Remember, no radicals can be left in the denominator.*

1. $\dfrac{2}{\sqrt{3}}$

6. $\sqrt{32}$

2. $\sqrt{50} \cdot \sqrt{10}$

7. $\sqrt{16} \cdot \sqrt{25}$

3. $\sqrt{98x^3 y^6}$

8. $\sqrt{56a^2 b^4 c^5}$

4. $\sqrt{\dfrac{81}{49}}$

9. $\dfrac{\sqrt{10p^3}}{\sqrt{27}}$

5. $\dfrac{4}{5+2\sqrt{3}}$

10. $\dfrac{3\sqrt{5}}{2-\sqrt{2}}$

Exponents

Rules of Exponents
• $a^m \cdot a^n = a^{m+n}$
• $(ab)^m = a^m b^m$
• $a^0 = 1$
• $(a^m)^n = a^{mn}$
• $\frac{a^m}{a^n} = a^{m-n}$
• $a^{-m} = \frac{1}{a^m}$

Ex. 1 $\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2} \cdot 2} = a^1 = a$

Ex. 2 $x^3 x^8 = x^{11}$

Ex. 3 $\left(\frac{x^2}{y}\right)^{-3} = \left(\frac{y}{x^2}\right)^3 = \frac{y^3}{x^6}$

Simplify the following. Remember there should be no negative exponents.

1. $-4x^5 y^{-2}$

2. $\frac{a^{-2}b^3}{c^{-4}d^{-1}}$

3. $(6x^3)^2$

4. $\left(\frac{2x}{3y^2}\right)^3$

5. $\frac{3^5}{3^3}$

6. $(7a^3 b^{-1})^0$

7. $\frac{1}{2^{-4}}$

8. $\frac{52x^6}{13x^{-7}}$

Multiplying Polynomials

FOIL: To multiply two binomials, find the sum of the products of the:

F – first terms
O – outer terms
I – inner terms
L – last terms

$(x - 2)(x + 4)$

$(x)(x) + (x)(4) + (2)(x) + (-2)(4)$

F   O   I   L

$x^2 + 4x - 2x - 8 = x^2 + 2x - 8$

Double-Distribution: Distribute both of the terms in the first parenthesis to the terms in the second parenthesis.

$(x - 2)(x+4)$

$x(x+4) - 2(x+4)$

$x^2 + 4x - 2x - 8 = x^2 + 2x - 8$

Find each product.

1. $(n + 8)(n + 2)$

2. $(y + 4)(y - 3)$

3. $(x - 3)(x+3)$

4. $(k + 12)(3k - 2)$

5. $(4h + 5)(h + 7)$

5. $(5m - 6)(5m - 6)$

Factoring

Factoring is used to represent quadratic equations in the **factored form** of $a(x - p)(x - q) = 0$, and solve this equation.

Factoring GCF

In a quadratic equation you may factor out the Greatest Common Factor.

Ex. 1: $16x^2 + 8x = 0$ GCF $= 8x$
$8x(2x + 1) = 0$ Zero Product Property

$8x = 0$ OR $2x + 1 = 0$
$x = 0$ OR $x = -1/2$

Factoring when a = 1

Ex. 2: $x^2 + 9x + 20 = 0$ To factor we want to find two numbers that multiply to be 20 and add to be 9.

$(x + 5)(x + 4) = 0$ $5 + 4 = 9$ and $5*4 = 20$

$(x + 5) = 0$ OR $(x + 4) = 0$ Zero Product Property

$x = -5$ or $x = -4$.

Solve each equation.

1. $20x^2 + 15x = 0$ 4. $6x^5 + 18x^4 = 0$

2. $x^2 - 16x + 64 = 0$ 5. $x^2 - 11x + 30 = 0$

3. $x^2 - 4x - 21 = 0$ 6. $x^2 - 6x - 16 = 0$

Factoring functions where a > 1 involves a different process. Some will use guess and check. Below is a method that will always work.

$ax^2 + bx + c$

Ex. 2: $6x^2 + 13x - 5 = 0$ Multiply a*c → (6)(-5) = -30

      -2, 15   Find two numbers that multiply to be – 30 and add to be 13.

      $6x^2 - 2x + 15x - 5 = 0$ Group the terms

      $(6x^2 - 2x) + (15x - 5) = 0$ Factor out GCF

      $2x(3x - 1) + 5(3x - 1) = 0$ Use the $(3x - 1)$ and factored terms

      $(2x + 5)(3x - 1) = 0$ Zero Product Property

      $2x + 5 = 0$ OR $3x - 1 = 0$ Solve

      x = -5/2 or x = 1/3

Solve the following by factoring.

7. $15x^2 - 8x + 1 = 0$ 8. $-12x^2 + 8x + 15 = 0$

Solving Quadratics Using the Quadratic Formula

Here are the steps required to solve a quadratic using the quadratic formula.

Solve: $x^2 - 8x + 14 = 0$

Step 1: Identify a, b, and c and plug them into the quadratic formula. In this case: a = 1, b = - 8, and c = 14.	**Step 1:** $x = \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(14)}}{2(1)}$
Step 2: Use the order of operations to simplify the quadratic function.	**Step 2:** $x = \frac{8\pm\sqrt{64-56}}{2} = \frac{8\pm\sqrt{8}}{2}$
Step 3: Simplify the radical, if you can. In this case you can simplify the radical into:	**Step 3:** $x = \frac{8\pm2\sqrt{2}}{2}$
Step 4: Reduce the problem, if you can. In this case you can reduce the entire problem by 2.	**Step 4:** $x = 4 \pm \sqrt{2}$

Now You Try:

Solve: $6x^2 - 13x - 8 = 0$

Graphing Quadratic Equations

To graph quadratic equations, identify the vertex by finding the x-value $x = \frac{-b}{2a}$ and plugging this x-value into the function to find the y-value. Remember, the standard for of Quadratics is y = ax^2+bx+c. Complete the table by filling in the grey section of the cart with the vertex and finding two points to the left and two points to the right of the vertex. Plot the points from the chart and connect the curve.

Tutorials:
Graphing Quadratic Equations (video)
https://www.khanacademy.org/math/algebra/quadratics/vertex-form-alg1/v/quadratic-functions-2
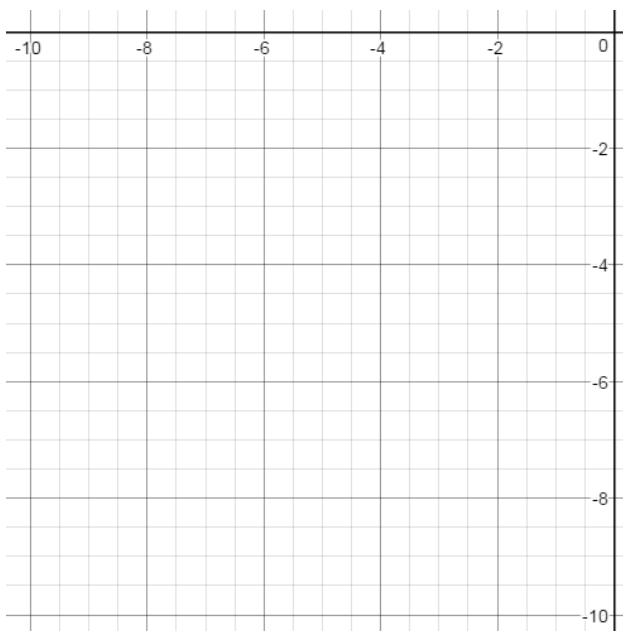
Graphing Quadratic Equations: http://www.purplemath.com/modules/grphquad.htm

For each equation, identify the vertex, fill in the chart, and graph the function:

1. $y = x^2 + 10x + 16$

Vertex: _____

X	Y

2. $y = -2x^2 - 4x - 3$

Vertex: _____

X	Y