# Mathematics Department 

# Summer Course Work 

in preparation for

## Calculus Honors

Completion of this summer work is required on the first day of the 2023-2024 school year.

# Bergenfield Public Schools <br> Mathematics Department <br> 80 South Prospect Avenue <br> Bergenfield, New Jersey <br> (201) 387-3850 

June 2023

Dear Parents and Guardians:

We are excited again to present summer activities that the math teachers of Bergenfield High School have created. Enclosed are math activities designed to help your son or daughter practice the skills which they have already learned and are critical to success in this course. As you may be aware, studies have shown that students who do not practice or review during the summer months the material they have already mastered lose some of that mastery. Unfortunately, this then requires the next teacher to spend valuable teaching time reviewing. While certainly not the final answer, this packet of activities is designed to help your son or daughter retain his or her math skills and knowledge.

Like you, we want your child to enjoy a wonderful summer. That is why we have designed activities so that 20 to 30 minutes of work per week should be all that is required. We urge you to encourage your child to take this task seriously and complete it successfully. Together we can make a difference in your child's future. Now is the time to build on the foundation to help your child succeed on future standardized exams, placement tests, and even more importantly, assessments at a college level.

These activities will reinforce skills that were taught in previous courses. This assignment should be completed and brought to the first day of the school in September. Calculators are NOT to be used to complete this project except where noted. Please read all directions carefully.

We wish you a wonderful and safer summer.

Sincerely,

Jim Fasano
Principal

Steven Neff
Supervisor of Mathematics

Skill A Writing an equation of a line in point-slope form
Recall The point-slope form for an equation of a line is $y-y_{1}=m\left(x-x_{1}\right)$.

## - Example

Write an equation for the line through $(1,-1)$ and $(-1,5)$
a. in point-slope form.
b. in slope-intercept form.

## - Solution

a. First find $m$.

$$
m=\frac{\text { difference in } y \text {-values }}{\text { difference in } x \text {-values }}=\frac{-1-5}{1-(-1)}=\frac{-6}{2}=-3
$$

Substitute the slope and one of the points into the point-slope equation.

$$
\begin{aligned}
y-y_{1} & =m\left(x-x_{1}\right) & & \\
y-(-1) & =-3(x-1) & & \text { Use the point }(1,-1) . \\
y+1 & =-3(x-1) & & \text { Simplify. }
\end{aligned}
$$

b. Rewrite the equation in the form $y=m x+b$.

$$
y+1=-3(x-1)
$$

$$
y+1=-3 x+3 \quad \text { Distributive Property }
$$

$$
y=-3 x+2 \quad \text { Subtract } 1 \text { from each side. }
$$

## Write an equation for each line in point-slope form.

7. containing $(4,-1)$ and with a slope of $\frac{1}{2}$ $\qquad$
8. crossing the $x$-axis at $x=-3$ and the $y$-axis at $y=6$ $\qquad$
9. containing the points $(-6,-1)$ and $(3,2)$ $\qquad$

## Rewrite each equation in slope-intercept form.

10. the line from Exercise 7 $\qquad$
11. the line from Exercise 8 $\qquad$
12. the line from Exercise 9 $\qquad$
13. In what situations would you find it easier to use point-slope form, and in what situations would you find it easier to use slope-intercept form? $\qquad$
$\qquad$
$\qquad$

Skill $\quad$ B actoring trinomials by choosing factor pairs of the constant
Recall Another way to factor a trinomial, such as $x^{2}-5 x-6$, is to first make a list of the pairs of factors of the constant. Then choose the right combination to complete the factors of the trinomial.

## - Example

Use the constant's factor pairs to factor $x^{2}-5 x-6$.

- Solution

List each pair of factors of -6 along with their sum.
Factors of $-6 \quad$ Sum of the factors
6 and $-1 \quad 5$

3 and -2 1
2 and $-3 \quad-1$
1 and $-6 \quad-5$
The sum of 1 and -6 is -5 . Use the combination of 1 and -6 to form the factors. Thus, $x^{2}-5 x-6=(x+1)(x-6)$.

Factor each trinomial. If the trinomial cannot be factored, write prime.
7. $x^{2}-x-2$
8. $x^{2}+3 x-4$
9. $x^{2}+4 x+3$
10. $x^{2}-4 x+3$
13. $x^{2}+2 x-15$
$\qquad$
16. $x^{2}+6 x+8$
11. $x^{2}+2 x-8$
12. $x^{2}+x-20$
15. $x^{2}-x-12$
18. $x^{2}+2 x-24$

Skill $\triangle$ Find the zeros of a polynomial function by factoring
Recall The zeros of a function are the values of $x$ that make $y$ equal to 0 .

## - Example 1

Find the zeros of the function $y=(x-2)(x+5)$.

## - Solution

Let $y=0$. Then use the Zero-Product Property to solve for $\boldsymbol{x}$.

$$
\begin{array}{rlrl}
(x-2)(x+5)=0 & & \\
(x-2)=0 & \text { or } & (x+5) & =0 \\
x=2 & \text { or } & x & =-5
\end{array}
$$

The zeros of $y=(x-2)(x+5)$ are 2 and -5 .
Recall A quadratic polynomial can be factored into two binomials.

## - Example 2

Solve the equation $x^{2}-x-6=0$.

## - Solution

Since $x^{2}-x-6$ can be factored into $(x+2)(x-3)$, you can rewrite $x^{2}-x-6=0$ as $(x+2)(x-3)=0$. Solve the equation $(x+2)(x-3)=0$.
$x+2=0 \quad$ or $\quad x-3=0$
$x=-2 \quad$ or $\quad x=3$
The solutions to $x^{2}-x-6=0$ are -2 and 3 .
Solve by factoring.
11. $x^{2}-4 x-12=0$
$\qquad$
13. $x^{2}-9 x+14=0$
$\qquad$
15. $x^{2}-7 x+10=0$
$\qquad$
17. $x^{2}+8 x+16=0$
$\qquad$
19. $9 x^{2}-1=0$
20. $4 x^{2}+4 x+1=0$

Skill D Using the quadratic formula to solve equations
Recall The solutions for a quadratic equation written in the form $a x^{2}+b x+c=0$, where $a \neq 0$, can be found by using the quadratic formula, $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

## - Example

Use the quadratic formula to solve $x^{2}-8 x+15=0$ for $x$.

- Solution

For $x^{2}-8 x+15=0, a$ is $1 ; b$ is -8 , and $c$ is 15 . Substitute these values in the quadratic formula.

$$
\begin{aligned}
x & =\frac{-(-8) \pm \sqrt{(-8)^{2}-(4)(1)(15)}}{(2)(1)} \\
& =\frac{8 \pm \sqrt{64-60}}{2} \\
& =\frac{8 \pm \sqrt{4}}{2} \\
& =\frac{8 \pm 2}{2} \\
x & =3 \text { or } x=5
\end{aligned}
$$

The solutions are 3 and 5 .

Use the quadratic formula to find the zeros of each function.
7. $y=x^{2}+2 x-8$
8. $y=2 x^{2}-x-15$
9. $y=4 x^{2}-8 x+3$

Skill E Using the discriminant to determine the number of solutions
Recall When a quadratic equation is written in the form $\mathrm{ax}^{2}+b x+c=0$, where $a \neq 0$, the expression $b^{2}-4 a c$ is called the discriminant of the quadratic formula.
If $b^{2}-4 a c>0$, there are two solutions.
If $b^{2}-4 a c=0$, there is one solution.
If $b^{2}-4 a c<0$, there are no real number solutions.

- Example

What does the discriminant tell you about $3 x^{2}-2 x+9=0$ ?

- Solution

For $3 x^{2}-2 x+9=0, a$ is $3 ; b$ is -2 , and $c$ is 9 .
Thus, $b^{2}-4 a c=(-2)^{2}-(4)(3)(9)=4-108=-104$
$-104<0$, so the equation $3 x^{2}-2 x+9=0$ has no real solutions.

Give the value of each discriminant. What does the discrimant tell you about the function?
10. $y=4 x^{2}+4 x+1$
11. $y=x^{2}+5 x+4$
12. $y=x^{2}+5 x+8$

Skill $F$ Writing and evaluating functions
Recall The value of $f(x)=x^{2}+5$ depends on the value of $x$.

## - Example 1

Sarah uses an internet server which charges $\$ 12.50$ per month plus $\$ 0.60$ for each hour over 20 hours that she uses it during the month. Write this relation in function notation. How much will she be charged for using the service for 38 hours in April?

## Solution

Let $h=$ number of hours over 20. Thus, the function is as follows.

$$
f(h)=12.50+0.60 h
$$

$$
f(18)=12.50+0.60(18) \quad \text { where } h=18
$$

$$
f(18)=23.30
$$

The charge for April will be $\$ 35.30$.

## Example 2

If $g(x)=x^{2}+3 x$, find $g(-5)$.

## - Solution

$g(-5)$ means replace $x$ with the value -5 and evaluate $g(x)$.

$$
\begin{aligned}
g(-5) & =(-5)^{2}+3(-5) \\
& =25-15 \\
& =10
\end{aligned}
$$

Thus, $g(-5)=10$.
Let $f(x)=5-\frac{2 x}{3}$ and $g(x)=\frac{1}{2} x^{2}+3 x$. Evaluate each function.
7. $f(6)$ $\qquad$ 8. $f(0)$
9. $f\left(\frac{1}{2}\right)$ $\qquad$
11. $g(-2)$ $\qquad$
13. $f(1)+g(0)$ $\qquad$
15. $f(0) \cdot g(0)$ $\qquad$

Skill G Using the four basic operations on functions to write new functions
Recall To write the sum, difference, product, or quotient of two functions, $f$ and $g$, write the sum, difference, product, or quotient of the expressions that define $f$ and $g$. Then simplify.

## - Example

Let $f(x)=x^{2}+3 x+2$ and $g(x)=5 x-1$. Write an expression for each function.
a. $(f+g)(x)$
b. $(f-g)(x)$
c. $(f g)(x)$
d. $\left(\frac{f}{g}\right)(x)$

## - Solution

a. $(f+g)(x)=f(x)+g(x)$

$$
=\left(x^{2}+3 x+2\right)+(5 x-1)
$$

$$
=x^{2}+8 x+1 \quad \text { Combine like terms. }
$$

b. $(f-g)(x)=f(x)-g(x)$

$$
\begin{aligned}
& =\left(x^{2}+3 x+2\right)-(5 x-1) \\
& =x^{2}+3 x+2-5 x+1 \quad \text { Combine like terms. } \\
& =x^{2}-2 x+3 \quad
\end{aligned}
$$

c. $(f g)(x)=f(x) \cdot g(x)$

$$
\begin{aligned}
& =\left(x^{2}+3 x+2\right)(5 x-1) \\
& =\left(x^{2}+3 x+2\right)(5 x)+\left(x^{2}+3 x+2\right)(-1) \quad \text { Distributive Property } \\
& =5 x^{3}+15 x^{2}+10 x-x^{2}-3 x-2 \\
& =5 x^{3}+14 x^{2}+7 x-2
\end{aligned}
$$

d. $\binom{f}{g}(x)=\frac{f(x)}{g(x)}$, where $g(x) \neq 0$

$$
=\frac{x^{2}+3 x+2}{5 x-1} \text {, where } x \neq \frac{1}{5}
$$

Let $f(x)=3 x^{2}+2, g(x)=2 x-1$, and $h(x)=x^{2}+5 x$. Find each new
function, and state any domain restrictions.

1. $(f+g)(x)$
$\qquad$
2. $(h-g)(x)$
$\qquad$
3. $(h g)(x)$
4. $\left(\frac{f}{g}\right)(x)$
$\qquad$
5. $(f+h)(x)$
6. $\left(\frac{h}{g}\right)(x)$

Skill H Finding the composite of two functions
Recall To write an expression for the composite function $(f \circ g)(x)$, replace each $x$ in the expression for $f$ with the expression defining $g$. Then simplify the result.

- Example

Let $f(x)=5 x$ and $g(x)=2 x^{2}-3$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$. Then write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

## - Solution

$(f \circ g)(2):$

$$
g(2)=2(2)^{2}-3=5
$$

$$
f(g(2))=f(5)=5(5)=25
$$

Thus, $(f \circ g)(2)=25$.

$$
(g \circ f)(2): \quad f(2)=5(2)=10 \quad g(f(2))=g(10)=2(10)^{2}-3=197
$$

Thus, $(g \circ f)(2)=197$.
To write expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$, use the variable $x$ instead of a particular number.

$$
\begin{aligned}
(f \circ g)(x) & =f(g(x)) & (g \circ f)(x) & =g(f(x)) \\
& =f\left(2 x^{2}-3\right) & & =g(5 x) \\
& =5\left(2 x^{2}-3\right) & & =2(5 x)^{2}-3 \\
& =10 x^{2}-15 & & =50 x^{2}-3
\end{aligned}
$$

Let $f(x)=x^{2}-1, g(x)=3 x$, and $h(x)=5-x$. Find each composite

## function.

9. $(f \circ g)(x)$
$\qquad$
10. $(h \circ f)(x)$
$\qquad$
11. $(g \circ g)(x)$
$\qquad$
12. $(g \circ h)(4)$
$\qquad$
13. $(f \circ(g \circ h))(1)$
$\qquad$

Recall A piecewise function in $x$ is a function defined by different expressions in $x$ on different intervals for $x$.

## - Example

Graph this piecewise function.

$$
f(x)= \begin{cases}|x|, & \text { if }-5 \leq x<-2 \\ {[x],} & \text { if }-2 \leq x<2 \\ 2 x-5, & \text { if } 2 \leq x \leq 5\end{cases}
$$

- Solution

| $x$ | -5 | -4 | -3 | -2.5 |
| :---: | ---: | ---: | ---: | ---: |
| $y=\|x\|$ | 5 | 4 | 3 | 2.5 |



| $x$ | -2 | -1.5 | -1 | -0.5 | 0 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=[x]$ | -2 | -2 | -1 | -1 | 0 | 1 |


| $x$ | 2 | 2.5 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y=2 x-5$ | 2 | 0 | 1 | 3 | 5 |

Graph each function.
15. $f(x)=\left\{\begin{array}{l}x+3, \text { if } x<0 \\ -2 x+5 \text { if } x \geq 0\end{array}\right.$

17. $f(x)=\left\{\begin{array}{l}|x| \text { if } x \leq 1 \\ 2-|x-2| \text { if } x>1\end{array}\right.$

16. $f(x)=\left\{\begin{array}{l}\frac{1}{2} x \text { if }-4 \leq x \leq 2 \\ 2 x-3 \text { if } x>2\end{array}\right.$

18. $f(x)=\left\{\begin{array}{l}{[x] \text { if }-2 \leq x \leq 1} \\ \lceil x\rceil \text { if } 1<x \leq 4\end{array}\right.$


Skill $\triangle \mathrm{J}$ Using logarithms to solve exponential equations
Recall The common logarithm, $\log _{10} x$, is usually written as $\log x$.

## - Example

Solve each equation.
a. $3^{x}=81$
b. $5^{x}=75$
c. $7^{x+1}=150$

## - Solution

a. $3^{x}=81$

Since 81 is a power of 3 , use powers of 3 .

$$
3^{x}=3^{4}
$$

$$
x=4 \quad \text { One-to-One Property of Exponential Functions }
$$

b. $5^{x}=75$

Since 75 is not a power of 5 , use logarithms to solve this equation.
$\log 5^{x}=\log 75$
$x \log 5=\log 75 \quad$ Power Property of Logarithms

$$
x=\frac{\log 75}{\log 5}
$$

$$
x \approx 2.68
$$

Check: $5^{2.68} \approx 75$
c. $\quad 7^{x+1}=150$
$\log 7^{x+1}=\log 150$
$(x+1) \log 7=\log 150$
$x+1=\frac{\log 150}{\log 7}$
$x=\frac{\log 150}{\log 7}-1$
$x \approx 1.57$
Solve each equation. Round your answers to the nearest hundredth.

1. $7^{x}=80$
$\qquad$
2. $4^{x+1}=100$
$\qquad$
3. $6^{2 x-7}=216$
4. $5^{3 x-1}=49$
5. $10^{x+5}=125$

Skill K Finding logarithms with bases other than 10
(You will need a calculator.)
Recall The equation $x=\log _{b} y$ is equivalent to $b^{x}=y$.

## - Example 1

Find $\log _{3} 40$ using your calculator.

## - Solution

Since calculators do not work in base 3, you can change this problem to a base 10 logarithm problem.

$$
\begin{array}{rlrl}
x=\log _{3} 40 \rightarrow 3^{x} & =40 & \\
\log 3^{x} & =\log 40 & & \text { base } 10 \text { logarithms } \\
x \log 3 & =\log 40 & & \\
x & =\frac{\log 40}{\log 3} & & \\
x & \approx 3.36 & & \text { Use a calculator. }
\end{array}
$$

Recall The change-of-base formula is $\log _{b} x=\frac{\log _{a} x}{\log _{a} b}$, where $a$ can be any permissible logarithmic base.

- Example 2

Find $\log _{5} 68$ using your calculator.

## - Solution

To find $\log _{5} 68$, use logarithms with base 10 . That is, use $a=10$. Then you can use a calculator's built-in base 10 logarithms.

$$
\begin{aligned}
x=\log _{5} 68 \Rightarrow \quad x & =\frac{\log 68}{\log 5} \\
x & \approx 2.62
\end{aligned}
$$

## Evaluate each logarithmic expression to the nearest hundredth.

10. $\log _{6} 18$
$\qquad$
11. $\log _{8} 512$
12. $\log _{10} 215$
13. $\log _{\frac{1}{2}} 24$
$\qquad$
14. $\log _{13} 110$
15. $\log _{2.5} 76$
16. $\log _{\frac{1}{16}} 329$

Skill LIdentifying parent functions in transformations
Recall Transformations of a function are indicated by the addition or subtraction of constants from the variable term or from the entire function or by multiplication or division of the variable term by a constant.

## - Example

In the following equations, identify the parent function:
a. $y=-3|x-2|+5$
b. $y=-(x+3)^{2}-4$

## Solution

a. Identify the additions, multiplications, subtractions, or divisions that occur. If the addition of 5 is removed, the equation becomes $y=-3|x-2|$. If the multiplication by -3 is removed, the equation becomes $y=|x-2|$. Finally, if the subtraction of 2 is removed, the equation becomes $y=|x|$. This is the absolute-value parent function.
b. Start with $y=-(x+3)^{2}-4$ and remove the additions and subtractions, starting with the subtraction of 4 outside the parentheses. This leaves $y=-(x+3)^{2}$. Then remove the negative sign preceding the parentheses, leaving $y=(x+3)^{2}$. Finally, remove the addition of 3 within the parentheses, producing the function $y=x^{2}$. This is the quadratic parent function.

## Identify the parent function for each of the following:

1. $y=-2|x+1|-4$ $\qquad$
2. $y=3(x-1)^{2}-2$ $\qquad$
3. $y=3 \cdot 2^{-x}+1$ $\qquad$
4. $y=-3(x+2)-4$ $\qquad$
5. $y=\frac{3}{x}+2$ $\qquad$
6. $y=\frac{3}{x+2}$
7. $y=3 x^{2}-4$ $\qquad$
8. $y=-2(x-1)^{2}$

Skill M Understanding the effect of order on combining transformations
Recall To determine the order of transformations to a function, reverse the order of operations. Addition or subtraction indicates a vertical translation; multiplication or division indicates a vertical stretch; addition or subtraction within parentheses or within absolute-value symbols indicates a horizontal translation.

## - Example

Describe the various transformations included in the equation $y=2|x-1|+3$.

## - Solution

The first operation to consider is the addition of 3 . This affects the parent function by translating it vertically 3 units up. The second operation, multiplication by 2 , stretches the translated function by a factor of 2 . The third operation, subtraction of 1 , translates the stretched function horizontally 1 unit to the right. Thus, the parent function, $y=|x|$, has been shifted 1 unit to the right, stretched by a factor of 2 , and then shifted 3 units up.

## Describe the transformations of the parent functions included in each equation.

9. $y=-3|x+2|-3$ $\qquad$
$\qquad$
$\qquad$
10. $y=2(x-3)^{2}+1$ $\qquad$
$\qquad$
$\qquad$
11. $y=4|x-1|+2$ $\qquad$
$\qquad$
$\qquad$
12. $y=4 \cdot 2^{x}-2$ $\qquad$
$\qquad$
$\qquad$

Skill N Solving 45-45-90 and 30-60-90 triangles

## Recall



## - Example

Find the lengths of the other 2 sides in each right triangle.
a.

b.


## - Solution

a. $a=5$
b. $a \sqrt{3}=6 \rightarrow a=\frac{6}{\sqrt{3}}($ or $2 \sqrt{3})$
$y=a=\frac{6}{\sqrt{3}}($ or $2 \sqrt{3})$
$r=a \sqrt{2}=5 \sqrt{2}$
$r=2 a=\frac{12}{\sqrt{3}}($ or $4 \sqrt{3})$

Find the missing side lengths in each right triangle.

2.

3.


Skill 0
Finding exact values of the trigonometric functions for an angle whose reference angle is $30^{\circ}, 45^{\circ}$, or $60^{\circ}$
Recall


| $\theta$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: |
| $\sin \theta$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos \theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $\tan \theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |

A reference angle is the positive acute angle between the terminal side of a given angle and the $x$-axis.

One mnemonic for remembering which functions are positive in each quadrant is "All students take calculus."

- Example

Find each exact value.
a. $\sin 315^{\circ}$
b. $\cos 240^{\circ}$
c. $\tan 210^{\circ}$

## - Solution

a. $315^{\circ}$ is in Quadrant IV, where sine is negative.
The reference angle is $45^{\circ}$.

$$
\begin{aligned}
\sin 315^{\circ} & =-\sin 45^{\circ} \\
& =-\frac{1}{\sqrt{2}}
\end{aligned}
$$

b. $240^{\circ}$ is in Quadrant III, where cosine is negative. The reference angle is $60^{\circ}$.

$$
\begin{aligned}
\cos 240^{\circ} & =-\cos 60^{\circ} \\
& =-\frac{1}{2}
\end{aligned}
$$




## Find each trigonometric value. Give exact answers.

4. $\sin 120^{\circ}$ $\qquad$
5. $\cos 330^{\circ}$ $\qquad$
6. $\tan 225^{\circ}$ $\qquad$
7. $\cos 150^{\circ}$ $\qquad$
8. $\sin 240^{\circ}$ $\qquad$
9. $\sin 150^{\circ}$ $\qquad$
10. $\tan 315^{\circ}$ $\qquad$
11. $\cos 225^{\circ}$ $\qquad$

Skill P Finding exact values for the trigonometric functions of an angle measured in radians

## - Example

Give the exact value of each expression where the angle measures are in radians.
a. $\cos \frac{5 \pi}{6}$
b. $\tan \frac{4 \pi}{3}$
c. $\sin \left(-\frac{3 \pi}{2}\right)$

## - Solution

Use reference angles.
a.

b.


$\frac{5 \pi}{6}$ radians $=150^{\circ}$
$\frac{4 \pi}{3}$ radians $=240^{\circ}$
$-\frac{3 \pi}{2}$ radians $=-270^{\circ}$
$\cos 150^{\circ}=-\cos 30^{\circ}=-\frac{\sqrt{3}}{2}$
$\tan 240^{\circ}=\tan 60^{\circ}=\sqrt{3}$
$\sin \left(-270^{\circ}\right)=\sin 90^{\circ}=1$

Evaluate each expression. Give exact answers.
16. $\sin \frac{3 \pi}{4}$ $\qquad$ 17. $\cos \frac{2 \pi}{3}$ $\qquad$ 18. $\tan \frac{5 \pi}{6}$ $\qquad$
19. $\cos \left(-\frac{7 \pi}{6}\right)$ $\qquad$
20. $\tan \left(-\frac{\pi}{4}\right)$ $\qquad$
21. $\sin \pi$ $\qquad$

Draw Each Angle in standard form below:
16.

17.

18.

19.

20.

21.


Skill Q Finding the trigonometric functions of an acute angle
Recall The hypotenuse is the longest side in a right triangle and is opposite the right angle.

- Example

Refer to the triangle shown at right and give values for $\sin \theta, \cos \theta, \tan \theta, \cot \theta, \sec \theta$, and $\csc \theta$.

## - Solution

The hypotenuse (hyp.) has a length of $\sqrt{41}$.
The leg opposite (opp.) $\theta$ has a length of 4 .
The leg adjacent (adj.) to $\theta$ has a length of 5 .

$\sin \theta=\frac{\text { opp. }}{\text { hyp. }}=\frac{4}{\sqrt{41}} \quad \csc \theta=\frac{\text { hyp. }}{\text { opp. }}=\frac{\sqrt{41}}{4}$
$\cos \theta=\frac{\text { adj. }}{\text { hyp. }}=\frac{5}{\sqrt{41}}$
$\sec \theta=\frac{\text { hyp. }}{\text { adj. }}=\frac{\sqrt{41}}{5}$
$\tan \theta=\frac{\text { opp. }}{\text { adj. }}=\frac{4}{5} \quad \cot \theta=\frac{\text { adj. }}{\text { opp. }}=\frac{5}{4}$
Refer to the triangle at right to find each value. Give exact answers.

1. $\sin \theta$ $\qquad$
2. $\cos \theta$ $\qquad$
3. $\tan \theta$ $\qquad$ 4. $\csc \theta$ $\qquad$
4. $\sec \theta$ $\qquad$
5. $\cot \theta$ $\qquad$

6. $\sin \phi$ $\qquad$
7. $\cos \phi$ $\qquad$
8. $\tan \phi$ $\qquad$
9. $\csc \phi$ $\qquad$
10. Draw Right Triangle ABC given the following properties:

$$
\begin{aligned}
& B C=20 \\
& A B=12
\end{aligned}
$$

12. Find each value (make sure to simplify your ratio)
a. $\sin B=$
b. $\cos \mathrm{B}=$
c. $\tan \mathrm{B}=$
d. $\quad \sin C=$
e. $\cos C=$
f. $\tan C=$

- Skill $R$ Applying inverse trigonometric functions

Recall $\sin \theta=\frac{\text { opp. }}{\text { hyp. }} \quad \cos \theta=\frac{\text { adj. }}{\text { hyp. }} \quad \tan \theta=\frac{\text { opp. }}{\text { adj. }}$

- Example

At a certain time of the day, the 5 meter flagpole shown at right casts a shadow that is 3 meters long.
What is the angle of elevation of the sun at this time?

- Solution

Since 3 meters is the length of the side adjacent to $\theta$ and 5 meters is the length of the side opposite $\theta$, use the tangent function.
$\tan \theta=\frac{5}{3}$
$\theta=\tan ^{-1}\left(\frac{5}{3}\right)$


This last equation states that $\theta$ is the angle that has a tangent of $\frac{5}{3}$.
$\theta \approx 59^{\circ} \quad$ Use calculator in degree mode.

## Find the measure of each angle to the nearest whole degree.

13. Find the measure of the smallest angle in a right triangle with sides of 3,4 , and 5 centimeters.
14. What is the angle between the bottom of the ladder and the ground as shown at right?
$\qquad$

15. Find the angle at the peak of the roof as shown at right.

16. The hypotenuse of a right triangle is 3 times as long as the shorter leg.

Find the measure of the angle between the shorter leg and the hypotenuse.

## Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x -value for which the function is undefined. That will be the vertical asymptote.
(a) Determine the vertical asymptotes for the function.
(b) Determine the x and y -intercepts
(1) $f(x)=\frac{1}{x^{2}}$
(2) $f(x)=\frac{x^{2}}{x^{2} \quad 4}$
(3) $f(x)=\frac{2+x}{x^{2}(1 \quad x)}$

## Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.
Case $\mathbf{I}$. Degree of the numerator is less than the degree of the denominator. The asymptote is $\mathrm{y}=0$.
Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)
(a) Determine all Horizontal Asymptotes
(b) Determine the $x$ and $y$-intercepts
(4) $f(x)=\frac{x^{2}-2 x+1}{x^{3}+x-7}$
(5) $f(x)=\frac{5 x^{3}-2 x^{2}+8}{4 x-3 x^{3}+5}$
(6) $f(x)=\frac{4 x^{5}}{x^{2}-7}$

