Bergenfield High School
Bergenfield, New Jersey

Mathematics Department

Summer Course Work

in preparation for

Financial Algebra

Completion of this summer work
is required on the first day of the
2023-2024 school year.

Student Name:________________________________

Bergenfield High School
Mathematics Department

Summer Workbook
College Prep Math
Topics

1. Multiplying Polynomials
2. Factoring
3. Quadratic Formula
4. Algebraic Expressions
5. Linear Equations
6. Exponents
7. Ordered Pairs/Graphing
8. System of Linear Equations
9. Square Roots and Simplifying Radicals

All pages MUST show the work in order for the work to be accepted. If more paper is needed, the work may go on the back of each page or neatly on a separate page.

Completion of this booklet is required by the first day of the school year.

***If you don't remember something, look it up. i.e. youtube, google, or Algebra 2 summer packet for examples.

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Dear Parents and Guardians:

Attached are the summer curriculum review materials for *College Prep Math*. This booklet was prepared by the Bergenfield High School Math department and contains topics that reflect content learned in prerequisite courses. These materials must be completed and brought to class on the first day of school in September.

Thank you for your cooperation.

Sincerely,

Jim Fasano	Steven Neff
Principal	Supervisor of Mathematics

Multiplying Polynomials

FOIL: To multiply two binomials, find the sum of the products of the: F – first terms O – outer terms I – inner terms L – last terms $(x-2)(x+4)$ $\underset{F}{(x)(x)} + \underset{O}{(x)(4)} + \underset{I}{(2)(x)} + \underset{L}{(-2)(4)}$ $x^2 + 4x - 2x - 8$ $x^2 + 2x - 8$	**Double-Distribution**: Distribute both of the terms in the first parenthesis to the terms in the second parenthesis. $(x-2)(x+4)$ $x(x+4) - 2(x+4)$ $x^2 + 4x - 2x - 8$ $x^2 + 2x - 8$

Find each product.

1. $(n+8)(n+2)$

2. $(y+4)(y-3)$

3. $(x-3)(x+3)$

4. $(k+12)(3k-2)$

5. $(4h+5)(h+7)$

6. $(5m-6)(5m-6)$

Factoring

Factoring is used to represent quadratic equations in the **factored form** of $a(x - p)(x - q) = 0$, and solve this equation.

Factoring GCF

In a quadratic equation you may factor out the Greatest Common Factor.

Ex 1: $16x^2 + 8x = 0.$ GCF = 8x

$8x(2x + 1) = 0$ Zero Product Rule

$8x = 0$ or $2x + 1 = 0$

$x = 0$ or $x = -1/2$

$$ax^2 + bx + c$$

Factoring where a = 1

Ex 2: $x^2 + 9x + 20 = 0$ To Factor we want to find two numbers that multiply to 20 and add to 9.

$(x + 5)(x + 4) = 0$ $5 + 4 = 9$ and $5 * 4 = 20$

$(x + 5) = 0$ *or* $(x + 4) = 0$ Zero Product Rule

$x = -5$ *or* $x = -4$

Solve each equation .

1. $20x^2 + 15x = 0$

2. $6x^5 + 18x^4 = 0$

3. $x^2 - 16x + 64 = 0$

4. $x^2 - 11x + 30 = 0$

5. $x^2 - 4x - 21 = 0$

6. $x^2 - 6x - 16 = 0$

Factoring functions where a > 1 involves a different process. Some will use guess and check. Below is a method that will always work.

$$ax^2 + bx + c$$

Ex 2: $6x^2 + 13x - 5 = 0$	Multiply a*c. $6(-5) = -30$
- 2, 15	Find two numbers that multiply the – 30 and sum to 13
$6x^2 - 2x + 15x - 5 = 0$	Group the Terms
$(6x^2 - 2x) + (15x - 5) = 0$	Factor GCF
$2x(3x - 1) + 5(3x - 1) = 0$	Use the (3x – 1) and factored terms
$(2x + 5)(3x - 1) = 0$	Zero Product Rule
$2x + 5 = 0$ *or* $3x - 1 = 0$	Solve
x = -5/2 or x = 1/3	

Solve the following by factoring.

7. $15x^2 - 8x + 1 = 0$

8. $-12x^2 + 8x + 15 = 0$

Solving Quadratics Using the Quadratic Formula

Here are the steps required to solve a quadratic using the quadratic formula.

Solve: $x^2 - 8x + 14 = 0$

Step 1: Identify a, b, and c and plug them into the quadratic formula. In this case $a = 1$, $b = -8$, and $c = 14$.	$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$
Step 2: Use the order of operations to simplify the quadratic formula.	$x = \frac{8 \pm \sqrt{64 - 56}}{2} = \frac{8 \pm \sqrt{8}}{2}$
Step 3: Simplify the radical, if you can. In this case you can simply the radical into:	$x = \frac{8 \pm 2\sqrt{2}}{2}$
Step 4: Reduce the problem, if you can. In this case you can reduce the entire problem by 2.	$x = 4 \pm \sqrt{2}$

Now You Try:

Solve: $6x^2 - 13x - 8 = 0$

Section 0.4: Algebraic Expressions

PEMDAS – the order in which you evaluate expressions
P – Parenthesis
E – Exponents
M – Multiplication **(from left**
D – Division **to right)**
A – Addition **(from left**
S – Substraction **to right)**

Evaluate each expression if $a = 2, b = -3, c = -1$, and $d = 4$.

1. $2a + c$

2. $\frac{bd}{2c}$

3. $\frac{2d-a}{b}$

4. $\frac{3b}{5a+c}$

Evaluate each expression if $x = 2, y = -3$, and $z = 1$.

5. $24 + |x - 4|$

6. $13 + |8 + y|$

7. $|5 - z| + 11$

8. $|2y - 15| + 7$

Section 0.5: Linear Equations

Example: Solve the equation.

$$\frac{2}{3}n + 1 = 11$$

$\begin{array}{r} \frac{2}{3}n + 1 = 11 \\ \underline{\quad -1 \quad -1} \end{array}$	Subtract 1 from each side.
$(3)\frac{2}{3}n = 10(3)$	Multiply each side by 3.
$\frac{2n}{2} = \frac{30}{2}$	Divide each side by 2.

Solve each equation.

1. $r + 11 = 3$
2. $\frac{8}{5}a = -6$
3. $c - 14 = -11$
4. $b + 2 = -5$
5. $5t = 30$
6. $\frac{m}{10} + 15 = 21$
7. $9n + 4 = 5n + 18$
8. $-2y + 17 = -13$
9. $-2(n + 7) = 15$

Exponents

Rules of Exponents

- $a^m \cdot a^n = a^{m+n}$
- $(ab)^m = a^m b^m$
- $a^0 = 1$
- $(a^m)^n = a^{mn}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $a^{-m} = \dfrac{1}{a^m}$

Ex. 1 $\left(a^{\frac{1}{2}}\right)^2 = a^{\frac{1}{2}\cdot 2} = a^1 = a$

Ex. 2 $x^3 x^8 = x^{11}$

Ex. 3 $\left(\dfrac{x^2}{y}\right)^{-3} = \left(\dfrac{y}{x^2}\right)^3 = \dfrac{y^3}{x^6}$

Simplify the following. Remember there should be no negative exponents.

1. $-4x^5y^{-2}$

2. $\dfrac{3^5}{3^3}$

3. $\dfrac{a^{-2}b^3}{c^{-4}d^{-1}}$

4. $\left(7a^3b^{-1}\right)^0$

5. $\left(6x^3\right)^2$

6. $\dfrac{1}{2^{-4}}$

7. $\left(\dfrac{2x}{3y^2}\right)^3$

8. $\dfrac{52x^6}{13x^{-7}}$

Section 0.7: Ordered Pairs

Important Notes:

- Points in the coordinate plane are known as **ordered pairs**.
- Ordered pairs are written in the form (x, y).
- The x-axis and y-axis divide the coordinate plane into four quadrants.
- The point of intersection of the axes is the **origin**.
- The origin is located at $(0,0)$.

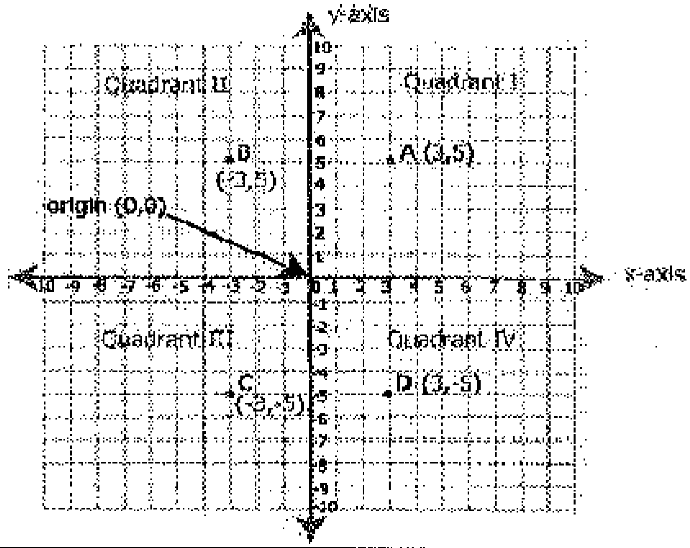
Example:

$A\,(3,5)$ - Quadrant I

$B(-3,6)$ - Quadrant II

$C(-3,-5)$ - Quadrant III

$D(3,-5)$ - Quadrant IV



Write the ordered pair for each point shown in the coordinate plane. Name the quadrant that the point is located in.

1. B = (,) **2.** C = (,)

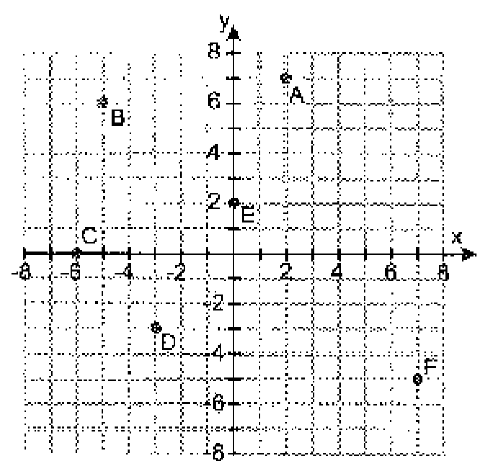
Quadrant = ________ Quadrant = ________

3. E = (,) **4.** A = (,)

Quadrant = ________ Quadrant = ________

5. D = (,) **6.** F = (,)

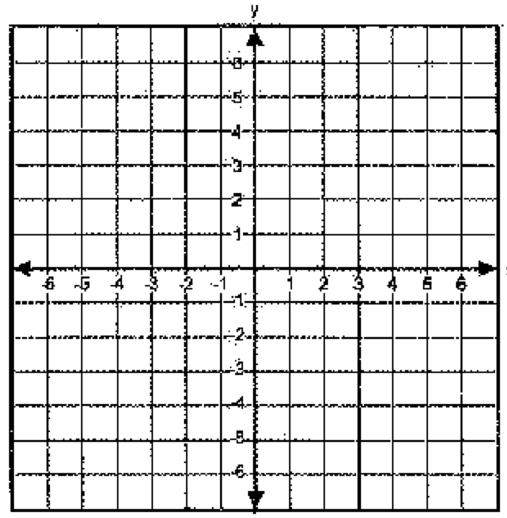
Quadrant = ________ Quadrant = ________



Make a table of values and graph 5 points that satisfy each equation.

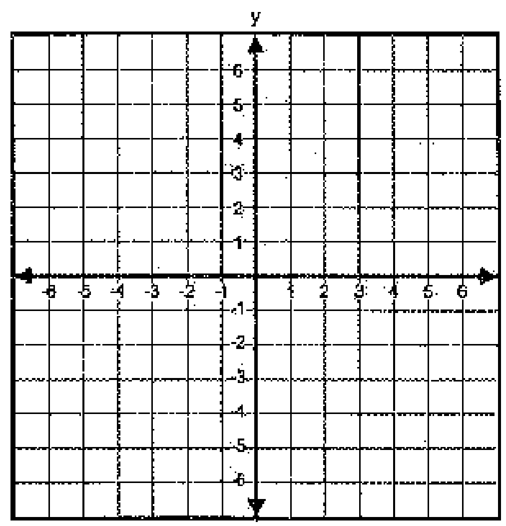
7. $y = 2x$

x	y



8. $y = 4 - x$

x	y

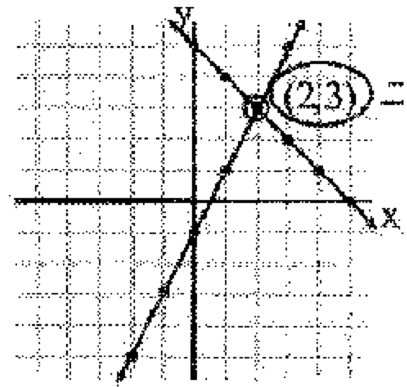


Section 0.8: Systems of Linear Equations

Solving Systems by Graphing

Example: Solve the system of equations by graphing.

$$y = 2x - 1$$
$$x + y = 5$$



Step 1: Graph each line. The second equation here needs to be changed to $y = mx + b$.

$$\begin{aligned} x + y &= 5 \\ -x \quad\; & -x \\ y &= -x + 5 \end{aligned}$$

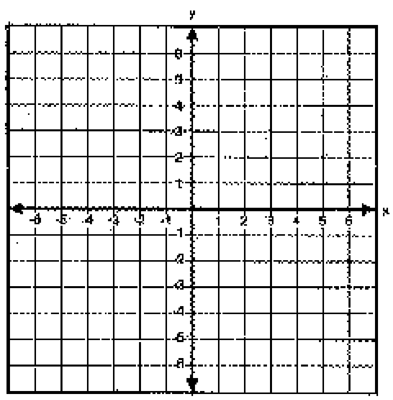
Step 2: Find the point of intersection.
If the lines overlap – infinitely many solutions
If lines are parallel – no solution

Solution: $(2, 3)$

Solve by graphing.

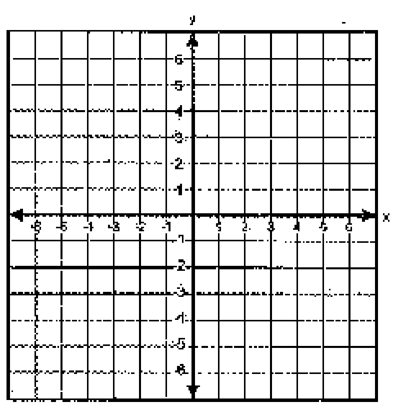
1. $y = -x + 2$

   $y = -\frac{1}{2}x + 1$



2. $y - 2x = 1$

   $2y - 4x = 1$



Solving Systems by Substitution

Example: Solve the system of equations by substitution.

$y - 3x = -3$
$-2x - 4y = 26$

Step 1: Solve for a variable for either equation. (It is ideal to pick the variable with a coefficient of 1)
$y - 3x = -3$
$\underline{\quad +3x \quad + 3x}$
$y = 3x - 3$

Step 2: Plug the expression $3x - 3$ in for y of the OTHER equation.
$-2x - 4y = 26$
$-2x - 4(3x - 3) = 26$

Step 3: Solve for x.
$-2x - 4(3x - 3) = 26$
$-2x - 12x + 12 = 26$
$-14x + 12 = 26$
$-14x = 14$
$x = -1$

Step 4: Plug in x for either equation to solve for y.
$y = 3x - 3$
$y = 3(-1) - 3$
$y = -6$

Final Solution: $(-1, -6)$

Solve by substitution.

3. $-5x + 3y = 12$
$x + 2y = 8$

4. $x - 4y = 22$
$2x + 5y = -21$

Solving Systems by Elimination

Example: Solve the system of equations by elimination.

$4x - 3y = 25$
$-3x + 8y = 10$

Step 1: Decide which variable you want to eliminate and find the LCM of the two coefficients for that variable.

Eliminate x → 4 and -3 have an LCM of 12

Step 2: Multiply each equation by the number that will make the x-terms have a coefficient of 12. One must be negative and the other must be positive.

$3(4x - 3y = 25)$ → $12x - 9y = 75$
$4(-3x + 8y = 10)$ $-12x + 32y = 40$

Step 3: Add the columns of like terms.

$$\begin{aligned} 12x - 9y &= 75 \\ \underline{-12x + 32y} &\underline{= 40} \\ 23y &= 115 \\ y &= 5 \end{aligned}$$

Step 4: Plug in y for either equation to solve for x.

$4x - 3y = 25$
$4x - 3(5) = 25$
$x = 10$

Final Solution: $(10, 5)$

Solve by elimination.

5. $-3x + y = 7$
$3x + 2y = 2$

6. $-4x + 5y = -11$
$2x + 3y = 11$

0.9: Square Roots and Simplifying Radicals

- **Product Property** for two numbers, $a,b \geq 0$, $\sqrt{ab} = \sqrt{a}\cdot\sqrt{b}$
- **Quotient Property** for any numbers a and b, where $a,b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Ex. 1 Simplify $\sqrt{45} = \sqrt{9\cdot 5} = \sqrt{3\cdot 3\cdot 5} = 3\sqrt{5}$

Ex. 2 Simplify $\sqrt{20x^3y^5z^6} = \sqrt{2^2\cdot 5\cdot x^3\cdot y^5\cdot z^6}$
$= \sqrt{2^2}\cdot\sqrt{5}\cdot\sqrt{x^3}\cdot\sqrt{y^5}\cdot\sqrt{z^6}$ $= 2\sqrt{5}\cdot x\sqrt{x}\ y^2\sqrt{y}\cdot z^3 = 2xy^2\sqrt{5xy}$

Ex. 3 Simplify $\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

Ex. 4 Simplify $\frac{2}{3-\sqrt{3}} = \frac{2}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{6+2\sqrt{3}}{9-\left(\sqrt{3}\right)^2} = \frac{6+2\sqrt{3}}{6} = \frac{3+\sqrt{3}}{3}$

Simplify the following radicals. *Remember, no radicals can be left in the denominator.*

1. $\frac{2}{\sqrt{3}}$

2. $\sqrt{32}$

3. $\sqrt{50}\cdot\sqrt{10}$

4. $\sqrt{16}\cdot\sqrt{25}$

5. $\sqrt{98x^3y^6}$

6. $\sqrt{56a^2b^4c^5}$

7. $\sqrt{\frac{81}{49}}$

8. $\frac{\sqrt{10p^3}}{\sqrt{27}}$

9. $\frac{4}{5+2\sqrt{3}}$

10. $\frac{3\sqrt{5}}{2-\sqrt{2}}$